

# Interacting Holographic Dark Energy model in Brans-Dicke cosmology and coincidence problem

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(Dated: January 25, 2017)

We study the dynamics of interacting holographic dark energy model in Brans-Dicke cosmology for the future event horizon and the Hubble horizon cut-offs. Typically, for the future event horizon cut-off, we determine the system of first-order differential equations and obtain the corresponding fixed points, attractors, repellers and saddle points. Finally, we investigate the cosmic coincidence problem in this model for the future event horizon and Hubble horizon cut-offs and find that for both cut-offs and for a variety of Brans-Dicke parameters the coincidence problem is almost resolved.

Keywords: Brans-Dicke parameter, entropy correction, holographic dark energy

PACS numbers: 98.80.-k; 95.36.+x; 04.50.Kd.

## I. INTRODUCTION

The observations type Ia Super Novae (SNeIa), Cosmic Microwave Background radiation (CMB) and Large Scale Structure (LSS) propose that the expansion of universe is accelerating [1]. The acceleration of the universe explains that the present universe is dominated by a mysterious form of energy with negative pressure so called dark energy (DE). The simplest candidate for dark energy is the cosmological constant. The cosmological constant suffers from the cosmic coincidence and fine-tuning problems [2, 3]. The cosmic coincidence problem expresses that why the dark energy density and matter density are of order unity [4]. A probable way to alleviate the cosmic coincidence problem is to suppose that there is an interaction between dark energy and matter. Also, the cosmic coincidence problem can be alleviated by suitable choice of the form of the interaction between dark energy and matter [5–7]. The nature of dark energy is unknown and mysterious. Therefore, people have proposed various models for dark energy such as: Quintessence, Tachyon [8], Ghost [9], K-essence, Phantom, Quintom, Chaplygin gas and Holographic [2, 3, 10]. Recently, the holographic dark energy (HDE) model based on the holographic principle was suggested by the following form of energy density [11]

$$\rho_{\Lambda} = 3c^2 M_P^2 L^{-2}, \quad (1)$$

where  $c$ ,  $M_P$  and  $L$  are the numerical constant, the reduced Planck mass and the cut-off radius, respectively.

One of the most important problems in cosmology is the coincidence problem. The coincidence problem arises by the question that “Why the ratio of dark matter density to dark energy density is the order of unity”? [12]. The holographic dark energy model is one of the proposed models to solve this problem in Einstein gravity. In this model, it has been proven that the interaction of cold dark matter with holographic dark energy can solve the coincidence problem [13, 14]. Motivated by finding a solution to the coincidence problem in the cosmological context of alternative gravity theories other than Einstein gravity, we investigate the coincidence problem in the interacting holographic dark energy model in the context of Brans-Dicke

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cosmology. In this regard, we obtain the equation of state parameter (EoS) for the cases of future event horizon and Hubble horizon cut-offs. We point out that the study of holographic dark energy model, considering the cosmological constant problem, almost rule out the Hubble horizon and particle horizon cut-offs due to the contradiction with observations, and just keep the future event horizon cut-off which is more consistent with observations [19]. Therefore, we focus on the future event horizon cut-off and determine the system of first-order differential equations and obtain the corresponding fixed points, attractors, repellers and saddle points, for this cut-off. Finally, we investigate the coincidence problem for the interacting holographic dark energy model with the future event horizon cut-off in Brans-Dicke cosmology, and just for comparison, we also study the coincidence problem for the interacting holographic dark energy model with Hubble horizon cut-off.

## II. INTERACTING LOGARITHMIC ENTROPY-CORRECTED HOLOGRAPHIC DARK ENERGY MODEL FOR FUTURE EVENT HORIZON CUT-OFF IN BRANS-DICKE COSMOLOGY

The action of Brans-Dicke theory in the canonical form can be written as [17, 18, 20]

$$S = \int d^4x \sqrt{g} \left( -\frac{1}{8\omega_{BD}} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right), \quad (2)$$

where  $g$ ,  $\omega_{BD}$ ,  $R$  and  $L_M$  are the determinant of the tensor metric  $g^{\mu\nu}$ , the Brans-Dicke parameter, the Ricci scalar curvature and  $L_M$  the lagrangian of the matter, respectively. Variation of the action with respect to the metric  $g^{\mu\nu}$  and the Brans-Dicke scalar field  $\phi$  obtain

$$\phi G_{\mu\nu} = -8\pi T_{\mu\nu}^M - \frac{\omega_{BD}}{\phi} \left( \phi_{;\mu} \phi_{;\nu} - \frac{1}{2} g_{\mu\nu} \phi_{;\kappa} \phi^{;\kappa} - \phi_{;\mu;\nu} + \square \phi g_{\mu\nu} \right), \quad (3)$$

$$\square \phi = \frac{8\pi}{2\omega_{BD} + 3} T_{\kappa}^{M\kappa}, \quad (4)$$

where  $T_{\mu\nu}^M$  is the energy-momentum tensor of the matter fields. The Friedman- Robertson- Walker (FRW) universe is represented by

$$ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (5)$$

where  $a(t)$  is the scale factor and  $k$  is the curvature parameter. Now, using Eq. (5) and inserting in Eqs. (3) and (4), one can write the Friedmann equation

$$\frac{3}{4\omega_{BD}} \phi^2 \left( H^2 + \frac{k}{a^2} \right) - \frac{1}{2} \dot{\phi}^2 + \frac{3}{2\omega_{BD}} H \dot{\phi} \phi = \rho_m + \rho_\Lambda + \rho_r + \rho_b, \quad (6)$$

where  $H = \dot{a}/a$  is the Hubble parameter. Also  $\rho_m$ ,  $\rho_\Lambda$ ,  $\rho_r$  and  $\rho_b$  are the pressureless dark matter density, dark energy density, radiation density and baryons density, respectively.

We assume that there is an interaction between dark matter and the logarithmic entropy-corrected holographic model of dark energy as follows

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \quad (7)$$

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (8)$$

and the interaction term is as follows [13]

$$Q = 3H(\lambda_\Lambda \rho_\Lambda + \lambda_m \rho_m), \quad (9)$$

where  $\lambda_\Lambda$  and  $\lambda_m$  are the coupling constants. Also, we suppose that radiation and baryons have no interaction with the logarithmic entropy-corrected holographic model of dark energy, so that they obey the continuity equations

$$\dot{\rho}_r + 4H\rho_r = 0, \quad (10)$$

$$\dot{\rho}_b + 3H\rho_b = 0. \quad (11)$$

Also, we assume that the Brans-Dicke field behaves as  $\phi = a^n$  [21], thus one can write

$$\dot{\phi} = nH\phi, \quad \ddot{\phi} = (n^2 H^2 + n\dot{H})\phi. \quad (12)$$

A case of special interest is when  $n$  is small while  $\omega_{BD}$  is large, so that the product  $n\omega_{BD}$  results in a value of order unity [21]. The fractional energy densities are presented by

$$\Omega_m = \frac{4\omega_{BD}\rho_m}{3\phi^2 H^2}, \quad (13)$$

$$\Omega_\Lambda = \frac{4\omega_{BD}\rho_\Lambda}{3\phi^2 H^2}, \quad (14)$$

$$\Omega_r = \frac{4\omega_{BD}\rho_r}{3\phi^2 H^2}, \quad (15)$$

$$\Omega_b = \frac{4\omega_{BD}\rho_b}{3\phi^2 H^2}, \quad (16)$$

$$\Omega_k = \frac{k}{a^2 H^2}. \quad (17)$$

Using Eqs. (12), (13), (14), (15), (16) and (17) and inserting Eq. (6), we can rewrite the Friedmann equation as follows

$$1 + \Omega_k - \frac{2}{3}n^2\omega_{BD} + 2n = \Omega_m + \Omega_\Lambda + \Omega_r + \Omega_b. \quad (18)$$

Also, taking time derivative of Eq. (6) and using Eqs. (7), (8), (9), (10), (11), (12), (13), (14), (15), (16) and (17) we can obtain

$$\frac{\dot{H}}{H^2} = \frac{-\frac{9}{4}\left[(1 + \omega_\Lambda)\Omega_\Lambda + \Omega_m + \Omega_b\right] - 3\Omega_r - \frac{3}{2}\Omega_k(n - 1) - \frac{3n}{2} + n^3\omega_{BD} - 3n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}. \quad (19)$$

Using Eq. (9) and inserting in Eq. (7), one can obtain the equation of state parameter

$$\omega_\Lambda = -1 - \frac{\dot{\rho}_\Lambda}{3H\rho_\Lambda} - \frac{(\lambda_\Lambda\Omega_\Lambda + \lambda_m\Omega_m)}{\Omega_\Lambda}. \quad (20)$$

The future event horizon cut-off is given by

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_x^\infty \frac{dx}{aH}. \quad (21)$$

Now, taking time derivative of Eq. (21) and using Eq. (21) one can obtain

$$\dot{R}_h = HR_h - 1. \quad (22)$$

By considering  $L = R_h$  and inserting in Eq. (1), we can obtain the density of holographic dark energy for the future event horizon cut-off as follows

$$\rho_\Lambda = \frac{3c^2\phi^2}{4\omega_{BD}R_h^2}. \quad (23)$$

Using Eq. (23) and inserting in Eq. (14) we can write

$$\Omega_\Lambda = \frac{c^2}{H^2 R_h^2}. \quad (24)$$

Now, taking time derivative of Eq. (23) and using Eqs. (22), (23) and (24) and inserting in Eq. (20) we can obtain the equation of state parameter of the holographic dark energy model for the future event horizon cut-off as follows

$$\omega_\Lambda = -\frac{1}{3} - \frac{2n}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c} - \frac{(\lambda_\Lambda\Omega_\Lambda + \lambda_m\Omega_m)}{\Omega_\Lambda}. \quad (25)$$

Also, the Hubble horizon cut-off is considered as

$$L = H^{-1}. \quad (26)$$

Using Eq. (26) and inserting in Eq. (1), we can obtain the density of holographic dark energy for the Hubble horizon cut-off as follows

$$\rho_\Lambda = \frac{3c^2\phi^2 H^2}{4\omega_{BD}}. \quad (27)$$

Using Eq. (27) and inserting in Eq. (14) we can write

$$\Omega_\Lambda = c^2. \quad (28)$$

Now, taking time derivative of Eq. (27) and using Eqs. (27), (28) and inserting in Eq. (20) we can obtain the equation of state parameter of holographic dark energy model for the Hubble horizon cut-off as follows

$$\begin{aligned} \omega_\Lambda = & \left[ -1 - \frac{2}{3} \left( n + \frac{9 - 6n^2\omega_{BD} + 24n + 3\Omega_k(1+2n) - 4n^3\omega_{BD} + 12n^2}{6 + 12n - 4n^3\omega_{BD}} \right) \right. \\ & \left. + \frac{18 - 12n^2\omega_{BD} + 48n + 6\Omega_k(1+2n) - 8n^3\omega_{BD} + 24n^2}{9 + 18n - 6n^3\omega_{BD}} - \frac{(\lambda_\Lambda\Omega_\Lambda + \lambda_m\Omega_m)}{\Omega_\Lambda} \right] \\ & \left[ 1 - \left( \frac{6c^2}{6 + 12n - 4n^3\omega_{BD}} \right) \right]^{-1}. \end{aligned} \quad (29)$$

### III. COSMOLOGICAL DYNAMICS OF INTERACTING LECHDE MODEL IN BRANS-DICKE COSMOLOGY

It has been shown that the model with future event horizon cut off is more consistent with the observations in comparison to the other cut-offs like Hubble horizon cut off [22]. Therefore, in this section we specify the system of first-order differential equations for interacting HDE model in BD cosmology. Also we can obtain the corresponding fixed points, the attractors, repellers and saddle points, typically for the future event horizon cut-off.

Taking time derivative of Eq. (14) and using Eqs. (14), (19), (27) and  $\dot{\Omega}_\Lambda = H\dot{\Omega}_\Lambda$  we can obtain

$$\begin{aligned} \dot{\Omega}_\Lambda = \Omega_\Lambda \left[ \left( 1 - \frac{1}{HR_h} \right) \left( \frac{2c^2}{H^2 R_h^2 \Omega_\Lambda} - 4 \right) + \frac{2c^2 n}{H^2 R_h^2 \Omega_\Lambda} - 2n \right. \\ \left. + \frac{\frac{9}{2} \left[ (1 + \omega_\Lambda)\Omega_\Lambda + \Omega_m + \Omega_b \right] + 6\Omega_r + 3\Omega_k(n-1) + 3n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n} \right], \end{aligned} \quad (30)$$

where  $' = d/dx$ . Taking time derivative of Eq. (13) and using Eqs. (8), (9), (12), (13), (14), (19) and  $\dot{\Omega}_m = H\Omega'_m$  we can obtain

$$\begin{aligned} \Omega'_m &= \Omega_m \left( -3 + 3\lambda_m - 2n \right) + 3\lambda_\Lambda \Omega_\Lambda \\ &+ 2\Omega_m \left( \frac{\frac{9}{4} \left[ (1 + \omega_\Lambda) \Omega_\Lambda + \Omega_m + \Omega_b \right] + 3\Omega_r + \frac{3}{2} \Omega_k (n-1) + \frac{3n}{2} - n^3 \omega_{BD} + 3n^2}{\frac{3}{2} - n^2 \omega_{BD} + 3n} \right). \end{aligned} \quad (31)$$

Taking time derivative of Eq. (15) and using Eqs. (10), (12), (15), (19) and  $\dot{\Omega}_r = H\Omega'_r$  we can obtain

$$\Omega'_r = -\Omega_r(4 + 2n) + 2\Omega_r \left( \frac{\frac{9}{4} \left[ (1 + \omega_\Lambda) \Omega_\Lambda + \Omega_m + \Omega_b \right] + 3\Omega_r + \frac{3}{2} \Omega_k (n-1) + \frac{3n}{2} - n^3 \omega_{BD} + 3n^2}{\frac{3}{2} - n^2 \omega_{BD} + 3n} \right). \quad (32)$$

Taking time derivative of Eq. (16) and using Eqs. (11), (12), (16), (19) and  $\dot{\Omega}_b = H\Omega'_b$  we can obtain

$$\Omega'_b = -\Omega_b(3 + 2n) + 2\Omega_b \left( \frac{\frac{9}{4} \left[ (1 + \omega_\Lambda) \Omega_\Lambda + \Omega_m + \Omega_b \right] + 3\Omega_r + \frac{3}{2} \Omega_k (n-1) + \frac{3n}{2} - n^3 \omega_{BD} + 3n^2}{\frac{3}{2} - n^2 \omega_{BD} + 3n} \right). \quad (33)$$

Now, for the present time, using Eqs. (24), (25), and inserting in Eqs. (30), (31), (32) and (33) we obtain

$$\begin{aligned} \Omega'_\Lambda &= \Omega_\Lambda \left[ -2 + \frac{2\sqrt{\Omega_\Lambda}}{c} + \right. \\ &\left. - \frac{3\Omega_\Lambda}{2} - \frac{3\Omega_\Lambda^{\frac{3}{2}}}{c} + 3n\Omega_\Lambda - \frac{9}{2}(\lambda_\Lambda \Omega_\Lambda + \lambda_m \Omega_m) + \frac{9}{2} - 3n^2 \omega_{BD} + 12n + \frac{3\Omega_r}{2} - 2n^3 \omega_{BD} + 6n^2 \right] \\ &\quad \left. \frac{\frac{3}{2} - n^2 \omega_{BD} + 3n}{} \right], \end{aligned} \quad (34)$$

$$\begin{aligned} \Omega'_m &= \Omega_m \left[ -3 + 3\lambda_m - 2n + \frac{3\lambda_\Lambda \Omega_\Lambda}{\Omega_m} + \right. \\ &\left. - \frac{9\Omega_\Lambda}{2} + 3n\Omega_\Lambda - \frac{9}{2}(\lambda_\Lambda \Omega_\Lambda + \lambda_m \Omega_m) + \frac{9}{2} - 3n^2 \omega_{BD} + 12n + \frac{3\Omega_r}{2} - 2n^3 \omega_{BD} + 6n^2 \right] \\ &\quad \left. \frac{\frac{3}{2} - n^2 \omega_{BD} + 3n}{} \right], \end{aligned} \quad (35)$$

$$\begin{aligned} \Omega'_r &= \Omega_r \left[ -4 - 2n + \right. \\ &\left. - \frac{3\Omega_\Lambda}{2} - \frac{3\Omega_\Lambda^{\frac{3}{2}}}{c} + 3n\Omega_\Lambda - \frac{9}{2}(\lambda_\Lambda \Omega_\Lambda + \lambda_m \Omega_m) + \frac{9}{2} - 3n^2 \omega_{BD} + 12n + \frac{3\Omega_r}{2} - 2n^3 \omega_{BD} + 6n^2 \right] \\ &\quad \left. \frac{\frac{3}{2} - n^2 \omega_{BD} + 3n}{} \right], \end{aligned} \quad (36)$$

$$\begin{aligned} \Omega'_b &= \Omega_b \left[ -3 - 2n + \right. \\ &\left. - \frac{3\Omega_\Lambda}{2} - \frac{3\Omega_\Lambda^{\frac{3}{2}}}{c} + 3n\Omega_\Lambda - \frac{9}{2}(\lambda_\Lambda \Omega_\Lambda + \lambda_m \Omega_m) + \frac{9}{2} - 3n^2 \omega_{BD} + 12n + \frac{3\Omega_r}{2} - 2n^3 \omega_{BD} + 6n^2 \right] \\ &\quad \left. \frac{\frac{3}{2} - n^2 \omega_{BD} + 3n}{} \right]. \end{aligned} \quad (37)$$

Now, we discuss the dynamical system determined by the Eqs. (34), (35), (36) and (37) for  $\Omega \equiv (\Omega_\Lambda, \Omega_m, \Omega_r, \Omega_b)$ . We solve the dynamical system of equations and obtain their fixed points by the corresponding matrix of linearization. We can determine the dynamical character of the fixed points by using the sign of the real part of the eigenvalues. The real parts of their eigenvalues demonstrate that the cosmological solutions are attractor, repeller or saddle points [23]. When all of the eigenvalues are negative, the fixed point is called an attractor, when all of the eigenvalues are positive, the fixed point is called a repeller;

otherwise the fixed point is called a saddle point. We present the eigenvalues of dynamical system in table 1. Also, in the Eqs. (34), (35), (36) and (37) we consider  $\lambda_\Lambda = \lambda_m = b^2$  [24] and  $\zeta$  and  $\varepsilon$  are defined as follows

$$\zeta \equiv \frac{-\frac{9}{2}b^2(\Omega_m + \Omega_\Lambda) + \frac{9}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}, \quad (38)$$

$$\chi \equiv \frac{-\frac{9}{2}b^2(\Omega_m + \Omega_\Lambda) + \frac{9}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2 + \frac{3}{2}\Omega_r}{\frac{3}{2} - n^2\omega_{BD} + 3n}. \quad (39)$$

The dominated Dark Matter model is defined as the DM model, the dominated Baryons model is defined as the B model which shows the early universe, the dominated Radiation model is defined as the R model, the dominated interacting dark Matter and dark Energy model is defined as the ME model, and the dominated interacting dark Matter and dark Energy model in the presence of radiation model is defined as the DMR model. In table 2, we characterize the attractor, repeller and saddle point properties for the fixed points determined in table 1.

**Table 1.** Fixed Points and Eigenvalues.

Model	Coordinates	Eigenvalues
DM	$(0, \Omega_m, 0, 0)$	$\lambda_1 = -2 + \frac{\frac{9}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$ $\lambda_2 = -3 - 2n + \frac{\frac{9}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$ $\lambda_3 = -4 - 2n + \frac{\frac{9}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$
B	$(0, 0, 0, \Omega_b)$	$\lambda_1 = -2 + \frac{\frac{9}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$ $\lambda_2 = -3 - 2n + \frac{\frac{9}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$ $\lambda_3 = -4 - 2n + \frac{\frac{9}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$
R	$(0, 0, \Omega_r, 0)$	$\lambda_1 = -2 + \frac{6 - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$ $\lambda_2 = -3 - 2n + \frac{6 - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$ $\lambda_3 = -4 - 2n + \frac{\frac{15}{2} - 3n^2\omega_{BD} + 12n - 2n^3\omega_{BD} + 6n^2}{\frac{3}{2} - n^2\omega_{BD} + 3n}$
ME	$(\Omega_\Lambda, \Omega_m, 0, 0)$	$\lambda_1 = -2 + \frac{3\sqrt{\Omega_\Lambda}}{c} + \frac{3\Omega_\Lambda \left( -1 - \frac{5\sqrt{\Omega_\Lambda}}{2c} + 2n - \frac{3b^2}{2} \right)}{\frac{3}{2} - n^2\omega_{BD} + 3n} + \zeta$ $\lambda_2 = -3 - 2n + \frac{3\Omega_\Lambda \left( -\frac{1}{2} - \frac{\sqrt{\Omega_\Lambda}}{c} + n + \frac{3b^2}{2} \right)}{\frac{3}{2} - n^2\omega_{BD} + 3n} + \zeta$ $\lambda_3 = -4 - 2n + \frac{3\Omega_\Lambda \left( -\frac{1}{2} - \frac{\sqrt{\Omega_\Lambda}}{c} + n \right)}{\frac{3}{2} - n^2\omega_{BD} + 3n} + \zeta$ $\lambda_4 = -3 - 2n + \frac{3\Omega_\Lambda \left( -\frac{1}{2} - \frac{\sqrt{\Omega_\Lambda}}{c} + n \right)}{\frac{3}{2} - n^2\omega_{BD} + 3n} + \zeta$
MER	$(\Omega_\Lambda, \Omega_m, \Omega_r, 0)$	$\lambda_1 = -2 + \frac{3\sqrt{\Omega_\Lambda}}{c} + \frac{3\Omega_\Lambda \left( -1 - \frac{5\sqrt{\Omega_\Lambda}}{2c} + 2n - \frac{3b^2}{2} \right)}{\frac{3}{2} - n^2\omega_{BD} + 3n} + \chi$ $\lambda_2 = -3 - 2n + 3b^2 + \frac{3\Omega_\Lambda \left( -\frac{1}{2} - \frac{\sqrt{\Omega_\Lambda}}{c} + n \right) - \frac{9b^2\Omega_m}{2}}{\frac{3}{2} - n^2\omega_{BD} + 3n} + \chi$ $\lambda_3 = -4 - 2n + \frac{3\Omega_\Lambda \left( -\frac{1}{2} - \frac{\sqrt{\Omega_\Lambda}}{c} + n \right) - 3\Omega_r}{\frac{3}{2} - n^2\omega_{BD} + 3n} + \chi$ $\lambda_4 = -3 - 2n + \frac{3\Omega_\Lambda \left( -\frac{1}{2} - \frac{\sqrt{\Omega_\Lambda}}{c} + n \right)}{\frac{3}{2} - n^2\omega_{BD} + 3n} + \chi$

**Table 2.** Attractor, Repeller and Saddle points.

<i>Model</i>	<i>Repeller</i>	<i>Attractor</i>	<i>Saddle point</i>
<i>DM</i>	$n \geq \frac{1}{2}, \lambda_2 > 1$	$n \leq -\frac{1}{2}$	-----
<i>B</i>	$n \geq \frac{1}{2}, \lambda_2 > 1$	$n \leq -\frac{1}{2}$	-----
<i>R</i>	$n \geq 0, n\omega_{BD} \leq 3$	$n \leq -\frac{1}{2}, n\omega_{BD} \geq 3$	-----
<i>ME</i>	$\lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0$	$\lambda_1, \lambda_2, \lambda_3, \lambda_4 < 0$	-----
<i>MER</i>	$\lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0$	$\lambda_1, \lambda_2, \lambda_3, \lambda_4 < 0$	-----

#### IV. COINCIDENCE PROBLEM FOR INTERACTING HDE MODEL IN BRANS-DICKE COSMOLOGY

In this section, we study the coincidence problem for HDE model in Brans-Dicke cosmology. We suppose that the contributions of the density of radiation and the density of baryons are negligible, thus we can write the Friedmann equation as follows

$$\frac{3}{4\omega_{BD}}\phi^2\left(H^2 + \frac{k}{a^2}\right) - \frac{1}{2}\dot{\phi}^2 + \frac{3}{2\omega_{BD}}H\dot{\phi}\phi = \rho_m + \rho_\Lambda. \quad (40)$$

Using Eqs. (12), (14) and inserting in Eq. (40), we obtain

$$\rho_m = \frac{3\phi^2 H^2}{4\omega_{BD}} \left(1 + \Omega_k - \frac{2}{3}n^2\omega_{BD} + 2n - \Omega_\Lambda\right). \quad (41)$$

Now, we consider the ratio of the density of dark matter to the density of dark energy as follows

$$r = \frac{\rho_m}{\rho_\Lambda}. \quad (42)$$

Using (14), (41) and inserting in Eq. (42), we can obtain

$$r = \frac{1 + \Omega_k - \frac{2}{3}n^2\omega_{BD} + 2n}{\Omega_\Lambda} - 1. \quad (43)$$

Taking time derivative of Eq. (43) and using (7),(8), (9), (43) and also we assume  $\lambda_\Lambda = \lambda_m = b^2$  [24], we obtain

$$\dot{r} = 3b^2 H(1+r)^2 + 3H\omega_\Lambda r. \quad (44)$$

Using Eq. (43) and inserting in Eqs. (25) and (29), we obtain the equation of state parameters for the future event horizon and Hubble horizon cut-offs, respectively

$$\omega_\Lambda = -\frac{1}{3} - \frac{2n}{3} - b^2(1+r) - \frac{2\sqrt{1 + \Omega_k - \frac{2}{3}n^2\omega_{BD} + 2n}}{3c\sqrt{1+r}}, \quad (45)$$

$$\begin{aligned} \omega_\Lambda = & \left[ -1 - b^2(1+r) + \frac{18 - 12n^2\omega_{BD} + 48n + 6\Omega_k(1+2n) - 8n^3\omega_{BD} + 24n^2}{9 + 18n - 6n^3\omega_{BD}} \right. \\ & \left. + \left( \frac{2c^2(1+r)}{3 + 3\Omega_k - 2n^2\omega_{BD} + 6n} \right) \left( \frac{6n - 4n^4\omega_{BD} + 9 - 6n^2\omega_{BD} + 3\Omega_k(1+2n) + 48n^2 - 4n^3\omega_{BD}}{-6 - 12n + 4n^3\omega_{BD}} \right) \right] \\ & \left[ 1 - \left( \frac{6}{6 + 12n - 4n^3\omega_{BD}} \right) \left( \frac{6 + 6\Omega_k - 4n^2\omega_{BD} + 12n}{3(1+r)} - c^2 \right) \right]^{-1}. \end{aligned}$$

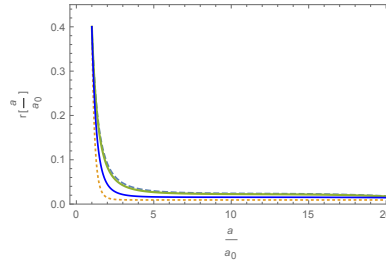


FIG. 1: The typical evolution of  $r(a/a_0)$  with respect to the scale factor  $a$  for the HDE model with the future event horizon cut-off ( $a_0$  denotes for the present value). The dashed line represents  $\omega_{BD} = 10000$ , the green line represents  $\omega_{BD} = 0$ , the black line represents  $\omega_{BD} = -100000$  and the dotted line represents  $\omega_{BD} = -500000$ .

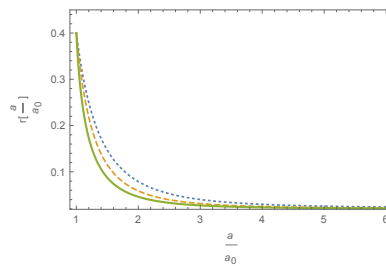


FIG. 2: The typical evolution of  $r(a/a_0)$  with respect to the scale factor  $a$  for the HDE model with the Hubble horizon cut-off ( $a_0$  denotes for the present value). The dotted line represents  $\omega_{BD} = 8000$ , the dashed line represents  $\omega_{BD} = 100000$ , the green line represents  $\omega_{BD} = -50000$ .

Now, for the present time we consider  $c^2 = 1.1$  [25],  $n = 0.005$  [26],  $\Omega_k = 0$ ,  $b^2 = 0.02$  [25] and using Eqs. (45), (46) and inserting in Eq. (44), we can plot  $r(a/a_0)$  in the figures (1) and (2) for the HDE model with the future event horizon and the Hubble horizon cut-offs. We find that for both cut-offs and for a variety of Brans-Dicke parameters, the fraction  $r(a/a_0)$  experiences a rather fast decreases at small scale factors, which means that at early stages of universe expansion the dark matter is transformed into dark energy in a rather high rate. However, for large scale factors the fraction  $r(a/a_0)$  approaches the almost constant values, which means that the interaction between dark energy and dark matter is frozen at late times and the transfer rate of dark matter to dark energy is almost vanishing. Therefore, we conclude that in this model of dark energy the coincidence problem is almost resolved.

## V. CONCLUDING REMARKS

In this work, we investigated the interacting holographic dark energy model in Brans-Dicke cosmology and obtained the equation of state parameter (EoS) for the cases of future event horizon and the Hubble horizon cut-offs. We determined the system of first-order differential equations and obtained the corresponding fixed points, attractors, repellers and saddle points for the future event horizon cut-off. We studied the coincidence problem for the interacting holographic dark energy model for the cases of future event horizon and the Hubble horizon cut-offs in Brans-Dicke cosmology. We found that for both cut-offs and for a variety of Brans-Dicke parameters, the coincidence problem is almost resolved.

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